Performance of Subcarrier Modulated Free-Space Optical Communications

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Abstract—Free-space optical communication is reputable for its ability to proffer solution to the access network bottle-neck but when used for long range communication, it suffers from scintillation caused by atmospheric turbulence. The probability of outage based performance of subcarrier intensity modulated free-space optical communication in log normal atmospheric turbulence is hereby presented. The bit-error-rate of subcarrier intensity modulated FSO employing phase shift keying (PSK) is also discussed. The atmospheric turbulence is modelled using log normal distribution and its effect mitigated by using an array of PIN photodetectors.

Key words—Free-space optical communication, turbulence, subcarrier modulation, PSK, spatial diversity.

I. Introduction
Free space optical (FSO) communications as an alternative means of providing high bandwidth over a short to medium range links has seen a growing increase in research and development activities over the past few years. Increasing commercial deployment of FSO could be said to be partly responsible for this surge in research activities [1]. FSO is used in a number of applications including the cellular communication backhaul, back-up link in optical fibre communications, exhibition halls, and disaster recovery among other emerging applications [1, 2]. However, of major concern in FSO is the dependence of its channel, the atmosphere, on the unpredictable weather conditions [3]. Effects of fog, rain, atmospheric gases and aerosols result in beam attenuation due to photon absorption and scattering [4, 5].

In dense fog conditions, laser radiations suffer prohibitive amount of attenuation limiting the FSO range to <5 m in such conditions [6]. However, in clear atmosphere with low extinction coefficient, a longer range is easily achievable. In clear weather conditions a primary factor that impairs the performance of FSO is the random changes in atmospheric temperature. The temperature fluctuations depend on the speed of wind and the height above the ground of the traversing laser radiation [4, 7-9]. The resulting eddies/cells of varying sizes (from ~ .1 cm to ~1 m) act like refractive prisms of varying index of refraction [9]. Consequently, a laser radiation traversing a turbulence atmosphere experiences random variation (fading) in its irradiance and phase. A familiar effect of turbulence is the twinkling of stars caused by random fluctuations of stars’ irradiance. The shimmer of the horizon on a hot day is another effect, this time caused by random changes in the optical phase of the light beam resulting in the reduced image resolution [9].

Techniques reported in literature to mitigate the effect of scintillation include forward error control [2, 1], spatial and temporal diversity [7, 11, 12], aperture averaging and adaptive optics [13-15]. In this paper, we will be considering subcarrier modulation with spatial diversity in log normal atmospheric turbulence. The spatial diversity scheme is advantageous in combating temporary link blockage by birds and misalignment when combined with wide laser beamwidth thereby eliminating the need for active tracking. It is also lots easier to provide independent aperture averaging with multiple separate aperture system, than in a single aperture where the aperture size has to be far greater than the irradiance spatial coherence distance [11]. Two different system performance metrics are presented; the generic bit-error-rate (BER) and outage probability $P_o$ which simply measures the probability that the BER is below a certain threshold level. The paper is arranged as follows: FSO system is briefly described in section 2, subcarrier modulation in section 3 while performance metrics and conclusion are given in sections 4 and 5, respectively.

II. FSO System Description
FSO like most communication systems has the following three functional elements:

A. The transmitter
This functional element has the primary function of converting the source information into optical radiation which is then propagated through the atmosphere to the receiver. The essential components of the transmitter are: the modulator in this case phase shift keying modulator (PSK); the driver circuit for the optical source which also helps stabilize the optical radiation against temperature fluctuations and component ageing; the optical source with direct intensity modulation (IM) where the radiation intensity/irradiance is proportional to the modulating signal [4, 5]; the transmitter telescope which collects, collimates and directs the optical radiation towards the receiver telescope via the atmospheric channel.
This is referred to as aperture averaging but a wide aperture 
radiations and focuses their average on the photodetector. (A 
large receiver telescope essentially collects and focuses the 
incoming optical radiation) an optical radiation traversing the 
medium resulting in random irradiance and phase variations. 
Detailed study of atmospheric turbulence can be found in [4, 7, 8, 17, 18]. In this work the log normal model of weak turbulence is 
considered with irradiation probability density function (pdf) 
given by [8]:

\[ p_I(I) = \frac{1}{I_0} \exp \left( -\frac{\ln(I/I_0)^2}{2\sigma^2} \right) \]

where \( I_0 \) represents the irradiance without turbulence, \( I \) is the 
received irradiance and \( \sigma^2 \) represents the log intensity 
variance (a measure of strength of irradiance fluctuation).

C. Receiver

This encompasses: (i) the receiver telescope which essentially 
collects and focuses the incoming optical radiation on to the 
photodetector. (A large receiver telescope aperture is advantageous 
as it collects verse uncorrelated radiations and focuses their 
average on the photodetector. This is referred to as aperture 
averaging but a wide aperture also means more background 
irradiation (noise)), (ii) an optical band-pass filter to reduce 
the background radiations, (iii) a photodetector, (iv) an 
amplifier, and a decision circuit.

Since, the atmospheric effects that degrade FSO performance 
are known to be wavelength independent [6]. FSO therefore uses 
the same wavelength as optical fibre communication to 
leverage on the availability and maturity of devices at such 
 wavelengths (785, 85 and 155 nm). In [19] it is reported that 
 improved availability can be achieved at 1 \( \mu \)m wavelength 
 but at the very high cost of devices.

III. Subcarrier Modulation

In optical subcarrier intensity modulation (SIM), an 
electrical signal (subcarrier) pre-modulated with the source 
data is used to modulate the intensity of an optical carrier. 
SIM is considered because it does not need adaptive 
threshold to perform optimally [2], it is resilient to the 
irradiance fluctuation and can be easily demodulated 
coherently by using well evolved low phase noise RF 
signal [21]. Figure 1 shows the block diagram of subcarrier 
intensity modulated FSO system. Prior to demodulating the 
laser irradiance, the subcarrier signal is pre-modulated with 
the source data \( d(t) \) using PSK. The 
subcarrier signal \( m(t) \) is DC level shifted to ensure that 
the bias current is always equal to or greater than the 
threshold current. Direct detection is employed at the 
receiver and the instantaneous photocurrent of the PIN 
photodetector is modelled as:

\[ i(t) = R I (1 + \xi m(t)) + n(t), \]  

where \( \xi \) is the modulation index, \( R \) is the photodetector 
responsivity. Over a symbol duration, the subcarrier signal 
is given by \( m(t) = A g(t) \cos(w_c t + \theta) \), \( g(t) \) represents 
the rectangular pulse shaping function, \( w_c \) is the subcarrier 
signal frequency, \( A \) is the subcarrier signal amplitude and 
\( n(t) \sim N(0, \sigma^2) \) is the additive noise. The modulation index \( \xi \) 
is chosen to keep the optical transmitter within its dynamic 
range and to avoid over-modulation \( \xi |m(t)| \leq 1 \). An estimate 
of the absolute phase of the RF subcarrier signal is then 
extracted from the photocurrent for subsequent coherent 
demodulation in order to recover the source data 
\( \hat{d}(t) \). In some cases, the phase extraction might be 
very demanding; in such situation differential phase shift 
keying (DPSK) should be adopted.

It is also possible to increase the system capacity by 
modulating different source data onto different RF 
subcarrier frequencies. The composite RF signal can then be 
used to modulate the irradiance of the optical source 
resulting in multiple SIM. Though, increased capacity is 
achieved but at a cost of increased transmitted optical
power. The analysis and performance of multiple SIM is outside the scope of this paper.

IV. Performance Metrics

Performance metrics are used to characterise, model and predict the behaviour of a system. In addition to the generic BER metric which models the fraction of bits received incorrectly in a communication system, we present an alternative metric—the outage probability due to turbulence. The outage probability is a measure of the probability that the instantaneous BER is less than a pre-determined threshold level. This metric adequately describes the system performance including short duration deep fades which the generic average BER does not. The BER and outage probabilities of SIM based FSO are discussed in the following sections.

A. Bit error rate (BER)

For an M-PSK modulated SIM optical communication through atmospheric turbulence where the channel state information is not known by the receiver, the BER (3) is obtained by averaging the BER of the M-ary PSK modulated RF subcarrier signal [22] over the atmospheric turbulence statistics.

\[ P_e \approx \frac{2}{\log_2 M} \int_0^\infty Q\left(\sqrt{\gamma \log_2 M \sin(\pi/M)}\right)p(I)\,dI, \quad (3) \]

where \( p(I) \) is the pdf of the received irradiance given by (1) for a lognormal distributed turbulence and \( \gamma = (RAI)^2/2\sigma^2 \) is the electrical SNR per bit at the input of the coherent demodulator. For DPSK based SIM, the BER expression adopted is given in [2]. Figure 2 shows the BER as a function of normalised SNR = \( (RI/E[I])^2/\sigma^2 \) for DPSK and M-ary PSK for log intensity variance = .52.

BPSK based SIM premised on coherent demodulation is apparently preferred for its superior error performance as elucidated in Fig. 2. However, if the phase can not be estimated from the received signal, DPSK based SIM can be used but at the expense of increased SNR. A compromise is therefore needed between simplicity and performance.

B. Outage probability

The FSO system performance with respect to average bit error rate has been well reported in [7, 2, 23]. However, a system with an adequate average bit error rate can temporarily suffer from increases in error rate due to deep fades and this ‘short outages’ is not adequately modelled by the average BER [24]. An alternative performance metric therefore is the outage probability due to turbulence, which is defined as the probability that \( P_e > P^*_e \), where \( P^*_e \) is the threshold BER.

This is akin to finding the probability that the \( \gamma \) that accounts for \( P_e \) is lower than the threshold SNR \( \gamma^* \); that is:

\[ P_o = P(P_e > P^*_e) = P(\gamma < \gamma^*). \quad (4) \]

\( \gamma^* = (RAI)^2/2\sigma^2 \) is the SNR with negligible atmospheric turbulence for a given level of background radiation. Parameter \( m \), here called power margin, is introduced to account for the extra power needed to cater for turbulence effect. The outage probability can then be derived from the combination of (1) and (4) as:

\[ P_o = P(m\gamma^* < \gamma^*) = Q(\ln m / \sigma_i - \sigma_i / 2). \quad (5) \]

Invoking the Chernoff upper bound \( Q(x) \leq .5\exp(-x^2/2) \) on (5) gives an upper bound on the outage probability from which the approximate additional power \( m \) needed to obtain \( P_o \) can be obtained as:

\[ m \approx \exp\left(\sqrt{-2\ln 2P_o\sigma_i^2 + \sigma_i^2 / 2}\right). \quad (6) \]

The amount of extra power needed to obtain a given outage probability is depicted in the Fig. 3 at various levels of irradiance fluctuation. To achieve an outage probability of 1-6, about 35 dBm of extra power is needed at \( \sigma_i^2 = .22 \). This will rise to ~43 dBm and ~48 dBm when the irradiance strength is \( \sigma_i^2 = .52 \) and \( \sigma_i^2 = .72 \), respectively. The extra margin is mainly due to the effect of turbulence and to reduce this, spatial diversity is employed. An array of N-PIN photodetectors is employed and the photocurrents \( \{I_i\}_i \) are linearly connected to obtain the overall received signal.

Fig. 2: BER against SNR for DPSK and M-ary-PSK for log intensity variance = .52. R and E[I] are both normalised to unity.
Fig. 3: Outage probability against power margin for various values of log intensity variance.

The spatial diversity is premised on the spacing of the photodetectors being greater than the optical radiation transverse coherence distance. This is to guarantee uncorrelated received irradiance so that the \(N\)-photodetectors do not all experience deep irradiance fades at the same time. It is also easier to achieve aperture averaging with a single detector with a very wide receiver aperture which might be so bulky and result in cumbersome design. To facilitate a fair comparison between single-transmitter-single-receiver-system and spatial diversity system, each pupil area in the \(N\)-photodetector system is assumed to be \(A_D/N\) where \(A_D\) is the detector area under single transmitter-single receiver link. It follows therefore that the background radiation noise \(\{n_i\}_i\) on each link with detector diversity is also reduced by a factor \(N\) from that with no spatial diversity resulting in \(\{n_i\}_i \sim N(\mu_u, \sigma_u^2 / N)\). By assuming identical PIN diode detector on each link, the individual detector output can be represented by:

\[ i_{ni}(t) = \frac{R}{N} I_i \left( 1 + A_g(t) \cos(\omega_c t + \theta) \right) + n_i(t), \quad (7) \]

where \(i = 1, 2, 3, \ldots, N\).

**Equal gain combining (EGC)**

In EGC, the diversity combiner shown in Fig. 4 collates the photocurrents \(\{i_{ni}\}_i\), extracts each subcarrier phase estimate and sums the outputs coherently with equal weight [25]. The combiner output photocurrent with suppressed D.C. component is given by:

\[ i_{EGC} = \sum_{i=1}^{N} \frac{R}{N} I_i \left( 1 + A_g(t) \cos(\omega_c t + \theta) \right) + n(t), \quad (8) \]

where \(\theta = \{ \frac{\pi}{2}, \pi \}\). The electrical SNR at the output of the EGC combiner conditioned on the received signal intensity can be derived as:

\[ \gamma_{EGC} = \frac{\left( RA \right)^2}{\sqrt{2N}} \left( \sum_{i=1}^{N} I_i \right)^2 \sigma^2 . \quad (9) \]

We assume that the sum of \(N\) lognormal random variables \(\sum_{i=1}^{N} I_i\) is another lognormal random variable \(Z = e^U\) where \(U \sim N(\mu_u, \sigma_u^2)\) [11]. The central limit theorem is not valid in this situation because the number of receivers under consideration is relatively small \((N \leq 1)\). The parameters of \(U \sim N(\mu_u, \sigma_u^2)\) are as defined in (1).

![Fig. 4. Spatial diversity receiver with \(N\)-detectors](image-url)
The exact outage probability and the upper bound at $\sigma_l^2 = .22$ for $N = 1$ and 4.

Figure 5 show how the upper bound of the outage probability (12) as a function of $m$ compares with the exact solution (11). The upper bound requires less than .5 dBm of power more than the exact solution and the upper bound appears to become tighter as the number of independent photodetector is increased. The gain due to spatial diversity is obtained as the ratio of (6) to (12).

$$Gain = \frac{1}{N} \exp \left( \sigma_l^2 / 2 + \mu_o - \sqrt{2 \sigma_l^2 \ln P_o} - \sqrt{-2 \sigma_o^2 \ln P_o} \right).$$

The plot of gain against $N$ at $P_o = 1 - 6$ is shown in Fig. 6 for various values of log intensity variance. From the plot, using two photodetectors result in about 4 dB gain at $\sigma_l^2 = .72$ and this rises to ~1 dB with $N \geq 6$. But at very low irradiance fluctuation ($\sigma_l^2 \leq .22$), increasing $N$ beyond 2 does not result in appreciable gain.

**V. Conclusion**

We have presented the performance of FSO in log normal modelled atmospheric turbulence based on BER and outage probability. In our analysis, subcarrier intensity modulation is employed and the effect of irradiance fluctuation is mitigated using spatial diversity. The gain of using spatial diversity has also been evaluated at various levels of turbulence and results presented.

**References**


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