Liverpool John Moores University
SCHOOL OF COMPUTING AND MATHEMATICAL SCIENCES

Semester 2 Examinations, May 2000

CMSMA1012 Further Mathematical Methods

Duration: 2 hours

Instructions to candidates

Do not open this question paper until you have been told to do so by the invigilator.

The figure in [] denotes the number of marks available for that question or part of question.

There are 6 questions. **Answer 4 questions.**

Questions carry 25 marks each. The total number of marks available is 100.

You have the use of the Software DERIVE 5™ and MATLAB. You may also use any Derive functions that you have developed throughout the course. You may not use a printer to print out any mathematical expressions or graphs.

Any results or graphs that DERIVE 5 produces, on screen, that are relevant to your examination work should be written or sketched in your answer book.

You may also use **any** calculator and its memory need **not** be erased before the examination.

The sending or the reading of email and use of the internet/intranet during this examination is prohibited.

1. (a) For the second order, constant co-efficient, linear, homogeneous equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 5y = 0$$

find

(i) the characteristic polynomial and its solutions
(ii) the general solution.

[15]

(b) Find the particular integral of the second order, constant co-efficient, linear, non-homogeneous equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 5y = e^{2x}$$

Hence write down the general solution to this differential equation.

[10]
Total [25]

2. (a) Given that $f(x, y) = x^3y + \sin(x^2y)$ evaluate

(i) f_x (ii) f_y (iii) f_{xx}

Also show that $f_{xy} = f_{yx}$ for the given example.

[8]

(b) You are given that $z = \cos x - \sin^2 y$ and that

$x = e^t$ and $y = \ln t$

Use the chain rule to find $\frac{dz}{dt}$ in terms of *t*.

[8]

(c) Find and classify the critical point of the function $z = x^2 - y^2 + 2x + 4y - 3$

[9]
Total [25]

3. (a) Given that $\underline{a} = \begin{pmatrix} 3 & 2 & 3 \end{pmatrix}^T$, $\underline{b} = \begin{pmatrix} 1 & 2 & -1 \end{pmatrix}^T$ and $\underline{c} = \begin{pmatrix} 0 & 1 & -1 \end{pmatrix}^T$

Evaluate

(i) $\underline{a}.\underline{b}$

(ii) $\underline{a}.\underline{c}$

(iii) $\underline{a}.(\underline{b} + \underline{c})$

[6]

(b) The lines L_1 and L_2 have the vector equations $\underline{r}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + s \begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix}$

and $\underline{r}_2 = \begin{pmatrix} 2 \\ 13 \\ -3 \end{pmatrix} + t \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$ respectively.

Determine the point of intersection of these two straight lines.

[7]

(c) The plane π passes through the points $A(1,1,0)$, $B(1,2,1)$ and $C(1,-1,0)$.

(i) Show that the vector $\overrightarrow{AB} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ and evaluate $\overrightarrow{AC}$

(ii) Evaluate $\overrightarrow{AB} \times \overrightarrow{AC}$, where $\times$ denotes cross product

(iii) Find the **vector** equation of the plane π

(iv) Using your answer to (iii) to write down the Cartesian equation of the plane

[12]

Total [25]

4. (a) Given that $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, evaluate *without using Derive*

(i) $\mathbf{A}^2$ (ii) $DET(\mathbf{A})$ (iii) $\mathbf{AB}$ (iv) $(\mathbf{AB})^T$

[8]

(b) Determine, using Derive or Matlab, the inverse of the matrix

$$\begin{pmatrix} 1 & 2 & 2 \\ 4 & 0 & -1 \\ 3 & 2 & 0 \end{pmatrix}$$

[2]

and describe how this inverse matrix can be used to solve the system of linear equations

$$\begin{aligned} x + 2y + 2z &= 1 \\ 4x \qquad - z &= 0 \\ 3x + 2y \qquad &= 2 \end{aligned}$$

[4]

and use the inverse matrix method to solve this system of linear equations.

[3]

(c) Find the characteristic equation of the matrix

$$\begin{pmatrix} 1 & 2 & 2 \\ 4 & 0 & -1 \\ 3 & 2 & 0 \end{pmatrix}$$

and hence find the eigenvalues of this matrix.

[8]

Total [25]

5. (a) Given that $z_1 = 1 + i$ and $z_2 = -1 + 2i$, evaluate *without using Derive*

(i) $z_1 z_2$ (ii) $z_1 \overline{z}_1$ (iii) $\arg(z_1 z_2)$

(iv) $\frac{z_1}{z_2}$ (v) $\arg\left(\frac{z_1}{z_2}\right)$

($\overline{z}$ represents the complex conjugate of the complex number *z*)

[10]

(b) Show that $e^{i\pi} = -1$.

[5]

(c) Use de Moivre's Theorem to show that

$$\sin(4\theta) = 4\sin\theta\cos\theta - 8\sin^3\theta\cos\theta$$

[10]

Total [25]

6. (a) Use the method of separation of variables to solve the first order differential equation

$$x(x^2 + 1)\frac{dy}{dx} = e^y$$

with the initial condition $x = 1, y = 0$

[12]

(b) Determine whether the differential equation

$$(3x + 2y^2) + 4xy\frac{dy}{dx} = 0$$

is an **exact** differential equation.

Hence solve the differential equation $(3x + 2y^2) + 4xy\frac{dy}{dx} = 0$.

[13]

Total [25]