

MA1011 Examination Answers 1998 Semester 1

Section A

Question 1

$$s := \{1, 2, \dots, 10\}$$

$$\text{COMPL}(a) := s \setminus a$$

$$a := \{2, 4, \dots, 10\}$$

$$b := \{1, 4, 9\}$$

$$\text{COMPL}(a \dot{\cup} b) = \{3, 5, 7\}$$

$$\text{COMPL}(a) \cdot \text{COMPL}(b) = \{3, 5, 7\} \quad [\text{M2}, \text{A3}]$$

$$\text{COMPL}(a \cdot b) = \{1, 2, 3, 5, 6, 7, 8, 9, 10\}$$

$$\text{COMPL}(a) \dot{\cup} \text{COMPL}(b) = \{1, 2, 3, 5, 6, 7, 8, 9, 10\} \quad [\text{M2}, \text{A3}]$$

Question 2

Shift -1 in the x direction [B1]

Scale/stretch by a factor of 3 parallel to the y axis [B2]

Shift -1 in the y direction [B2]

Question 3

$$F(n, i) := 1.0609^n \cdot i \quad [\text{M3}, \text{A3}]$$

$$F(20, 16000)$$

$$5.219260467 \cdot 10^4 \quad \{\text{A2}\}$$

$$F(n, 16000) = 24000$$

$$\text{SOLVE}(F(n, 16000) = 24000, n)$$

$$[n = 6.858618708]$$

7 years [M3, A4]

Question 4

(a) Must use the product rule

$$\frac{d}{dx} (x^2 \cdot \sin(3x)) = 3x^2 \cdot \cos(3x) + 2x \cdot \sin(3x)$$

[M2, A3]

$$\frac{d}{dx} (1+x)^2 = \frac{x \cdot \hat{e}}{(x+1)^2}$$

$$y \cdot \frac{d}{dx} (1+x)^2 = \frac{x \cdot y}{(x+1)^2}$$

[M2, A3]

Question 5

(a) $2\sin(x/2) + c$

[m1, a2]

(b)

$$\int x \cdot e^{3x} dx - \int \frac{e^{3x}}{3} dx$$

$$\frac{2 \cdot \hat{e}^3}{9} - \frac{1}{9}$$

[M3, A4]

Section B
Question 1

$$\text{FIT} \begin{array}{l} \text{€} \\ \text{„} \\ \text{...x, } a \cdot x^2 + b \cdot x + c \text{,} \\ \text{•} \end{array} \begin{array}{l} \text{„} \\ \text{0} \\ \text{2} \\ \text{3} \end{array} \begin{array}{l} \text{1} \\ \text{4} \\ \text{4} \end{array} \begin{array}{l} \text{†} \\ \text{†} \\ \text{†} \\ \text{†} \end{array} \begin{array}{l} \text{f} \\ \text{f} \\ \text{f} \\ \text{f} \end{array}$$

$$- \frac{x^2}{2} + \frac{5 \cdot x}{2} + 1$$

$$\text{„} \begin{array}{l} \text{†} \\ \text{†} \\ \text{†} \end{array} \begin{array}{l} \text{„} \\ \text{<33} \\ \text{5} \end{array} \begin{array}{l} \text{†} \\ \text{†} \\ \text{†} \end{array} \begin{array}{l} \text{„} \\ \text{2} \\ \text{2} \end{array} \begin{array}{l} \text{†} \\ \text{†} \\ \text{†} \end{array} \begin{array}{l} \text{„} \\ \text{5} \\ \text{<33} \end{array} \begin{array}{l} \text{†} \\ \text{†} \\ \text{†} \end{array} \begin{array}{l} \text{f} \\ \text{f} \\ \text{f} \end{array}$$

$$\int_0^{33/2 + 5/2} \left(-\frac{x^2}{2} + \frac{5 \cdot x}{2} + 1 \right) dx$$

$$\frac{11 \cdot <33}{8} + \frac{185}{24}$$

$$\text{FIT} \begin{array}{l} \text{€} \\ \text{„} \\ \text{...x, } a \cdot x^2 + b \cdot x + c \text{,} \\ \text{•} \end{array} \begin{array}{l} \text{„} \\ \text{0} \\ \text{2} \\ \text{3} \end{array} \begin{array}{l} \text{1} \\ \text{-2} \\ \text{-2} \end{array} \begin{array}{l} \text{†} \\ \text{†} \\ \text{†} \\ \text{†} \end{array} \begin{array}{l} \text{f} \\ \text{f} \\ \text{f} \\ \text{f} \end{array}$$

$$y = 1$$

$$\frac{x^2}{2} - \frac{5 \cdot x}{2} + 1$$

$$- \frac{x^2}{2} + \frac{5 \cdot x}{2} + 1 = \frac{x^2}{2} - \frac{5 \cdot x}{2} + 1$$

$$[x = 0, x = 5]$$

$$\int_0^5 \left(-\frac{x^2}{2} + \frac{5 \cdot x}{2} + 1 \right) dx$$

$$\frac{185}{12}$$

$$\frac{185}{12} - 5 = \frac{125}{12}$$

125/6 Area

Question 2B

(a)

$$\text{TAYLOR} \left\langle \left\langle \frac{\epsilon \epsilon 1 + 2 \cdot x}{1 + x} \right\rangle, x, 0, 5 \right\rangle$$

$$\frac{399 \cdot x^5}{256} - \frac{141 \cdot x^4}{128} + \frac{13 \cdot x^3}{16} - \frac{5 \cdot x^2}{8} + \frac{x}{2} + 1$$

[M2, A1]

(b)

$$\left\langle \left\langle \frac{\epsilon 1 + 2 \cdot 0.3}{1 + 0.3} \right\rangle, \right\rangle$$

1. 109400392

[M1, A1]

(c)

$$\frac{399 \cdot 0.3^5}{256} - \frac{141 \cdot 0.3^4}{128} + \frac{13 \cdot 0.3^3}{16} - \frac{5 \cdot 0.3^2}{8} + \frac{0.3}{2} + 1$$

1. 110552226

[M1, A1]

$$\{1. 110552226 - 1. 109400392\}$$

0. 001151834000

0. 00115

{M2, A1}

(d)

$$\text{TAYLOR} \left\langle \left\langle \frac{\epsilon \epsilon 1 + 2 \cdot x}{1 + x} \right\rangle, x, 0, 10 \right\rangle$$

1. 109370352

$$\{1. 109370352 - 1. 109400392\}$$

3. 003999999 · 10⁻⁵

[M2, A1]

$$\text{TAYLOR} \left\langle \left\langle \frac{\epsilon \epsilon 1 + 2 \cdot x}{1 + x} \right\rangle, x, 0, 15 \right\rangle$$

1. 109401631

$$\{1. 109401631 - 1. 109400392\}$$

1. 238999646 · 10⁻⁶

[M1, A1]

(e) Yes

[A1]

(f) Errors for n=5, 10, 15 are

[0. 1317281443, 0. 2310461936, 0. 6515150768]

so errors are increasing, therefore not converging

Question 3

(a) Use of a diagram and the correct expression

[M4, A3]

$$\sum_{r=1}^n \frac{1}{n} \cdot 3 \cdot (r/n) \cdot \hat{e}$$

[A3]

$$\frac{\sum_{r=1}^n \frac{3}{n} \cdot 3 \cdot (\hat{e} - 1)}{n \cdot (\hat{e} - 1)}$$

$$\lim_{n \rightarrow \infty} \frac{\sum_{r=1}^n \frac{3}{n} \cdot 3 \cdot (\hat{e} - 1)}{n \cdot (\hat{e} - 1)}$$

$$\frac{3 \cdot (\hat{e} - 1)}{3}$$

[M2, A2]

(b)

$$\text{width} := \frac{b - a}{n}$$

$$\text{height} := \hat{e} \cdot 3 \cdot (a + (b - a) \cdot r/n)$$

width · height

$$\frac{\sum_{r=1}^n \hat{e} \cdot 3 \cdot a \cdot (n - r)/n + 3 \cdot b \cdot r/n}{n} \cdot (b - a)$$

$$\sum_{r=1}^n \frac{\hat{e} \cdot 3 \cdot a \cdot (n - r)/n + 3 \cdot b \cdot r/n}{n} \cdot (b - a)$$

$$\frac{\sum_{r=1}^n \hat{e} \cdot 3 \cdot b/n \cdot (b - a) \cdot (\hat{e} - 1)}{n \cdot (\hat{e} - 1)}$$

$$\lim_{n \rightarrow \infty} \frac{\sum_{r=1}^n \hat{e} \cdot 3 \cdot b/n \cdot (b - a) \cdot (\hat{e} - 1)}{n \cdot (\hat{e} - 1)}$$

$$\frac{3 \cdot b}{3} - \frac{3 \cdot a}{3}$$

Question 4

(a) If $y=|x|$ then

$$\lim_{x \rightarrow 0^+} \frac{d}{dx} |x| = 1$$

$$\lim_{x \rightarrow 0^-} \frac{d}{dx} |x| = -1$$

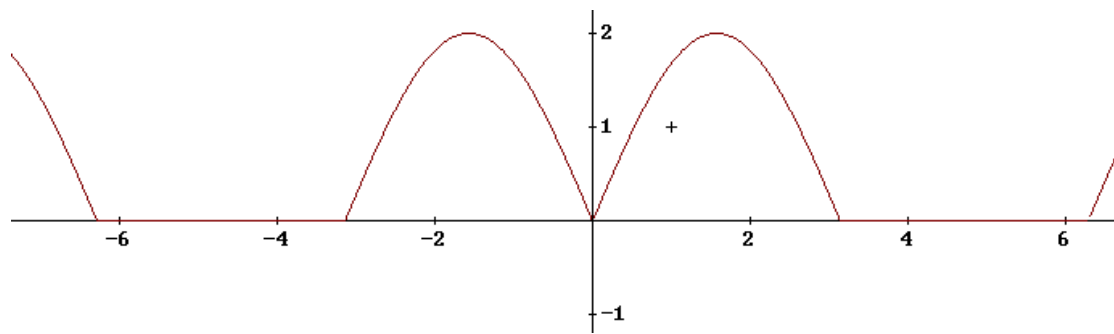
Hence not differentiable at $x=0$,

$$\lim_{x \rightarrow 0^-} |x| = 0$$

$$\lim_{x \rightarrow 0^+} |x| = 0$$

Hence cs at $x=0$

(b)



Not differentiable at $[-2 \cdot 1, -1, 0, 1, 2 \cdot 1]$

[B5]

(c)

$$\lim_{x \rightarrow 1^-} \frac{d}{dx} (|\sin(x)| + \cos(|x|))$$

-1

$$\lim_{x \rightarrow 1^+} \frac{d}{dx} (|\sin(x)| + \cos(|x|))$$

1

HENCE NOT DIFFERENTIABLE

[M3, A2]

$$F(x) := \frac{\ln(x^2 + 3 \cdot x + 1)}{\sin(x)}$$

$[F(-0.001), F(0.001)]$

$[3.003506512, 2.996506487]$

[M2, A1]

VECTOR("F(-10⁻ⁿ), F(10⁻ⁿ)", n, 1, 10)

3. 430617927846480693	2. 704777080487100779
3. 035662595613542016	2. 965637917802560015
3. 003506512359047484	2. 996506487692269431
3. 000350065012335899	2. 999650064987669231
3. 000035000650012333	2. 999965000649987666
3. 000003500006833346	2. 999996500006833319
3. 000000350000063333	2. 999999650000063333
3. 000000035000000408	2. 999999965000000408
3. 000000003500000004	2. 999999996500000004
3. 000000000300000000	2. 999999999625000000

(e)

$$\frac{F(x+h) - F(x)}{h}$$

$$\frac{\frac{\ln(x^2 + x(2h+3) + h^2 + 3h + 1)}{h \cdot \sin(x+h)} - \frac{\ln(x^2 + 3x + 1)}{h \cdot \sin(x)}}{h}$$

$$\lim_{h \rightarrow 0} \frac{\frac{\ln(x^2 + x(2h+3) + h^2 + 3h + 1)}{h \cdot \sin(x+h)} - \frac{\ln(x^2 + 3x + 1)}{h \cdot \sin(x)}}{h}$$

$$\frac{2x+3}{(x^2 + 3x + 1) \cdot \sin(x)} - \frac{\cos(x) \cdot \ln(x^2 + 3x + 1)}{\sin(x)^2}$$

$$\lim_{x \rightarrow 0} \frac{2x+3}{(x^2 + 3x + 1) \cdot \sin(x)} - \frac{\cos(x) \cdot \ln(x^2 + 3x + 1)}{\sin(x)^2}$$

$$-\frac{7}{2}$$

[M3, A2]