Liverpool John Moores University

SCHOOL OF COMPUTING AND MATHEMATICAL SCIENCES

Semester Examinations: December 1999

CMSMA1011 Mathematical Methods

Duration: 2 hours

INSTRUCTIONS TO CANDIDATES

There are two sections, **A** and **B**.

You must answer **all** questions in section **A** and **3** questions from section **B**.

You are advised not to spend more that 40 minutes on section **A**.

You have the use of a networked PC and the software Derive for Windows. In addition you may use any other software that is available to you at that time. However, you are **not** permitted to send emails or use any Internet browser.

You may also use **any** calculator of your choice and the memory need **not** be erased.

You may not print out any material during this examination.

Section A

Answer all questions. You are advised not to spend more than 30 minutes on this section.

1. The universal set S is the set of natural numbers from 1 to 10. You are given that $A = \{1,4,7,8\}$ and $B = \{4,9,10\}$. Using the sets A and B and the given universal set, demonstrate the validity of de Morgan's laws, i.e. that $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$ and $\overline{(A \cap B)} = \overline{A} \cup \overline{B}$. [10]

2. Describe the transformations of $y = \sin(x)$ that will produce

$$y = 3\sin(3(x - \pi)) + 1$$

[5]

3. An investment plan returns 5.01% interest per annum. Write down a function, in terms of n the number of years and I the amount invested, that will give the value of the investment plan after n completed years. Hence, evaluate the value of a £16,000 investment plan after 15 years at a fixed rate of 5.01% p.a. Find also, the minimum numbers of years it will take for the investment to increase by 20%

[15]

4. Using **pencil and paper methods only**, evaluate the following derivatives. Full marks can only be gained by showing full working.

   (a) $\dfrac{d}{dx} x^2 (1+x)^{20}$ (b) $y = \sqrt{(1+x^2)}$

[10]

5. Using **pencil and paper only**, evaluate the following integrals. Full marks can only be gained by showing full working.

   (a) $\int \cos(2x)\, dx$ (b) $\int_0^1 x e^{-x} dx$

[10]

Section B

Answer **3** questions

1. (a) Find the quadratic function, y_1, that passes through the points, $A(-1,1)$, $B(0,4)$ and $C(1,4)$. [6]

   (b) Find the area enclosed by y_1, the *x* axis and the *y* axis. [6]

   (c) Find the quadratic function, y_2, that is a reflection of y_1 in the line $y = 1$. [6]

   (d) Find the area enclosed by the functions y_1 and y_2 [7]

Total [25]

2. (a) Find the Taylor expansion, T_5, of $\frac{1-2\sin x}{1-x}$ of degree 5 about the point $x = 0$. [3]

   (b) Evaluate the function $\frac{1-2\sin x}{1-x}$ at the point $x = 0.3$. [2]

   (c) Evaluate T_5 at the point $x = 0.3$ and calculate the error (to 3 significant figures) in using T_5 to approximate $\frac{1-2\sin x}{1-x}$ at the point $x = 0.3$ [5]

   (d) Using the Taylor expansions of degree 10 & 15 respectively of the function $\frac{1-2\sin x}{1-x}$ about the point $x = 0$, find the error (to 3 significant figures) in using these approximations to evaluate $\frac{1-2\sin x}{1-x}$ at the point $x = 0.3$.
   [n.b. you do not need to write out the Taylor series.] [5]

   (e) Is the Taylor expansion of $\frac{1-2\sin x}{1-x}$ convergent at the point $x = 0.3$? [1]

   (f) Determine, with evidence, whether the Taylor expansion of $\frac{1-2\sin x}{1-x}$ about $x = 0$ is convergent at $x = 1.1$. [9]

Total [25]

3. (a) If the area under the curve $y = \sin(4x)$ between $x = 0$ and $x = \dfrac{\pi}{4}$ is approximated by n upper Riemann rectangles, show that the area of the r^{th} rectangle is

$$\frac{\pi}{4n}\sin\left(\frac{\pi r}{n}\right)$$ [7]

Hence write down an expression for the total area of the rectangles and evaluate this with Derive for Windows. [3]

From this expression, deduce $\int_0^{\frac{\pi}{4}} \sin(4\pi)\,dx$ [4]

(b) Use the method of upper Riemann rectangles to prove that

$$\int_a^b \sin(4x)\,dx = \frac{-\cos(4b)}{4} + \frac{\cos(4a)}{4}$$

[You must show the relevant steps to gain full marks] [11]

Total [25]

4. (a) Explain why $y = |\sin x|$ is not differentiable at $x = 0$, but is continuous at $x = 0$. [5]

(b) Find all the points at which $y = |\sin x| + \sin|x|$ is not differentiable in the region $-2\pi \le x \le 2\pi$. [7]

(c) Show that $y = |\sin|x|| + |\cos|x||$ is not differentiable at $x = \dfrac{\pi}{2}$. [5]

(d) It is suggested that $\lim_{x \to 0} \dfrac{\ln(x^2 + 4x + 1)}{\sin x} = 4$. It the computer algebra system you are using does not have a function for calculating limits, suggest a strategy for determining whether this limit is feasible. [3]

(e) Find, from first principles, the gradient of $\dfrac{\ln(x^2 + 4x + 1)}{\sin x}$ at the point $x = 0$. **[You must show the relevant steps to gain full marks]** [5]

Total [25]