Liverpool John Moores University

SCHOOL OF COMPUTING AND MATHEMATICAL SCIENCES

Semester Examinations December 1999

CMSCD1006 Abstraction & Modelling

Duration: 2 hours

INSTRUCTIONS TO CANDIDATES

There are 25 questions.

Answer ALL questions, indicating your answer to each question on the answer sheet at the end of this paper.

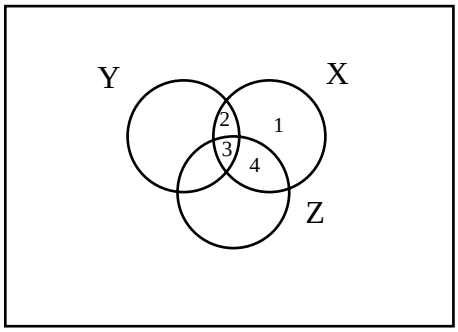
For each question, your answer should only be ONE of **A** or **B** or **C** or **D**.

Questions carry 4 marks each.

A sheet containing logic laws is provided.

Candidates should hand in both the answer sheet and any separate answer booklet containing any evidence of working.

1. In the Venn diagram below, which numbered region represents the set $(X \cap Z) \cap \overline{Y}$?



A: 1 **B:** 2 **C:** 3 **D:** 4

2. If X is a set containing 5 elements and Y is a set containing 7 elements, what is the minimum number of elements that $X \cup Y$ may contain?

A: 12 **B:** 2 **C:** 7 **D: 5**

3. Given that *X* and *Y* are non-empty sets and that $X \subset Y$, which one of the following is TRUE?

A: $X \cap \overline{Y} = \varnothing$ **B:** $X \cap Y = Y$ **C:** $X \cup Y = X$ **D:** $\overline{X} \cap Y = \varnothing$

4. If $X = \{a,b,c,d\}$, $Y = \{c,d,e,f\}$ and $Z = \{a,b,e,f\}$, which of the following is equal to the set $(X \cup Y) \setminus Z$?

A: $\{a,b,c,d\}$ **B:** $\{c,d\}$ **C:** $\{e,f\}$ **D:** $\varnothing$

5. Given that $X = \{1,2,3,4\}$, $Y = \{3,4,5,6\}$ and that $n(A)$ represents the number of elements in any set *A*, which one of the following is FALSE ?

A: $n(X) = 4$ **B:** $n(X \cup Y) = n(X) + n(Y)$
C: $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$ **D:** $n(X \cap Y) = 2$

6. *p, q, r* denote the propositions:

p : the engine is off *q* : the hand brake is on *r*: the car can move

Which one of the following symbolically represents the statement:

If the engine is on and the handbrake is off , then the car can move

A: $(p \wedge \neg q) \rightarrow r$ **B:** $(\neg p \wedge \neg q) \rightarrow \neg r$ **C:** $(\neg p \vee \neg q) \rightarrow r$ **D:** $(\neg p \wedge \neg q) \rightarrow r$

7. Which of the columns A, B, C, D correctly completes the truth table for the predicate $(p \wedge q) \rightarrow (p \vee q)$.

P	q	**A**	**B**	**C**	**D**
T	T	T	T	T	T
T	F	T	T	F	F
F	T	T	T	F	F
F	F	F	T	T	F

8. Which of the columns A, B, C, D correctly completes the truth table for the predicate $\neg p \vee (\neg q \wedge r)$

p	q	r	**A**	**B**	**C**	**D**
T	T	T	F	F	F	T
T	T	F	F	F	T	T
T	F	T	T	F	F	F
T	F	F	F	F	F	T
F	T	T	T	T	T	F
F	T	F	T	T	T	F
F	F	T	T	F	T	F
F	F	F	T	T	T	T

9. Is the predicate $\neg(\neg(p \wedge q) \rightarrow p)$ logically equivalent to

A: a tautology **B:** a contradiction **C:** $\neg p$ **D:** p.

10. Predicates $E(x)$, $F(x)$ and $N(x)$ are defined as

$E(x)$: x is an even number and positive, $F(x)$: $x < -5$, $N(x)$: $x < 0$.

x is restricted to belong to the set of negative integers. Which one of the following is true?

A: $\forall x \mid F(x) \vee N(x)$ **B:** $\exists x \mid F(x) \wedge E(x)$

C: $\forall x \mid F(x) \wedge N(x)$ **D:** $\exists x \mid \neg N(x)$

11. $S = \{1, 2, 3, 4, 5, 6\}$. The relation ***R*** is defined on the set S by

$aRb \Leftrightarrow a + b$ is divisible by 3

Consider the statements p : ***R*** is a reflexive relation,
q : ***R*** is a symmetric relation,
r : ***R*** is a transitive relation.

Which option is correct?

A: p, q, r are true **B:** Only q and r are true

C: Only r is true **D:** Only q is true.

12. S = {1, 2, 3} and T = {2, 3, 4, 5, 6}. A relation *R* is defined on *S*x*T* by

$$x\ R\ y \text{ iff } 3x = y$$

What is the complete set of ordered pairs that satisfy *R*?

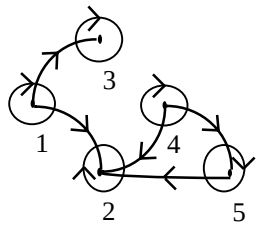
A: {(1, 3)} **B:** {(2, 6)} **C:** {(1,3), (2, 6)} **D:** {(1, 3), (3, 1), (2,6), (6,2)}

13. A relation *class_list* is defined as $class\_list = \{(n, s) \mid n \in students \wedge s \in subject\_list\}$
Where $students = \{Arthur, Brian, Colin, Denise, Ellen\}$ and $subject\_list = \{CD1006, GN1003, CB1004\}$

Which of the following is true?

A: $(Ellen, CB1004) \in class\_list$ **B:** $(CD1006, Colin) \in class\_list$
C: $(Brian, Colin, GN1003) \in class\_list$ **D:** $(Brian, GN1003) \notin class\_list$

14. A relationship R acting on the set S = {1, 2, 3, 4, 5} is described by the arrow diagram below. Which statement about R is true?



A: reflexive, symmetric but not transitive **B:** symmetric, transitive but not reflexive
C: an equivalence relation **D:** an order relation.

15. A relationship *R* is defined on S = {0,1, 2, 3, 4, 5} by *a*R*b* iff $a^2 \equiv b \pmod 6$

The number of equivalence classes into which *R* partitions S is

A: 1 **B:** 2 **C:** 3 **D:** 4

16. *f, g, h* are defined as:

f: $S \to T$,where $S = T = \{1, 2, 3\}$ and $f = \{(1, 1), (2, 3), (3, 1), (2, 1)\}$
g: $\mathbb{Z} \to \mathbb{N}$ $g(x) = |x|$ (the absolute value of *x*)
h: $\mathbb{N} \to \mathbb{N}$ $h(x) = \begin{cases} x+2 & \text{if } x \le 5 \\ x & \text{if } x \ge 5 \end{cases}$

Which of the above are functions?

A: *f*, not *g*, not *h* **B:** not *f*, *g*, not *h*
C: *f*, not *g*, *h* **D:** none of *f*, *g*, *h*.

17. Two functions *f* and g are defined by

$f : \mathbb{Z} \to \mathbb{N}, \ f(x) = x^2 + 1$

$g : \mathbb{N} \to \mathbb{Q}, g(x) = \dfrac{1}{(x+1)}$, where $\mathbb{N} = \{0, 1, 2, 3, \cdots\}$

Which is true?

A: *f* and *g* are both bijective **B:** neither *f* nor *g* is bijective

C: *f* is bijective, *g* is not **D:** *g* is bijective, *f* is not

18. Two functions *f* and *g* are defined by

$f : \mathbb{R} \to \mathbb{R}, \quad f(x) = 2x + 6$

$g : \mathbb{R} \to \mathbb{R}, \quad g(x) = \dfrac{x}{3}$

A function $h : \mathbb{R} \to \mathbb{R}$ is defined as $h(x) = gfg(x)$

What is $h(9)$?

A: 4 **B:** 8 **C:** 12 **D:** 3

19. *f* and *g* are partial functions defined from A to B, where:

A = {0, 1, 2, 3}, $B = \{x \mid x \in \mathbb{N} \ \wedge \ 4 \le x \le 10\}$

f: $x \mapsto x^2 + 1$ *g*: $x \mapsto 10 - 2x$

Which one of the following is true:

A: dom(*f*) = A **B:** ran(*g*) = B **C:** dom(*f*) ⊂ dom(*g*) **D**: ran(f)⊂ ran(g).

20. g: ACC ↦ BAL represents the table

Acc. No.	Balance
2345	12.99
2842	354.89
2891	460.88

Which table is represented by the expression $(\{2891\} \lhd (g \oplus \{2842 \mapsto 22.60\})) \cup \{2974 \mapsto -96.72\}$.

A:

Acc. No.	Balance
2345	12.99
2842	22.60
2974	-96.72

B:

Acc. No.	Balance
2345	12.99
2891	22.60
2974	-96.72

C:

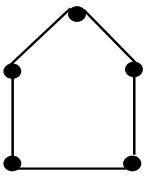
Acc. No.	Balance
2842	22.60
2974	-96.72

D:

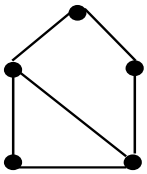
Acc. No.	Balance
2345	12.99
2842	22.60
2891	460.88
2974	-96.72

21. By definition, which graph is not the same as the others?

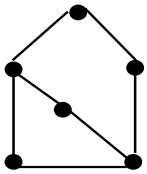
A:



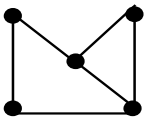
B:



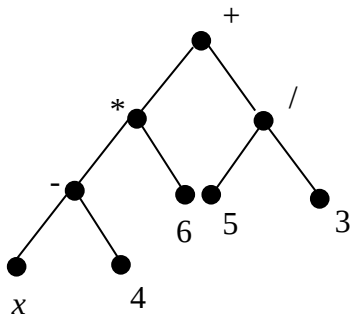
C:



D:

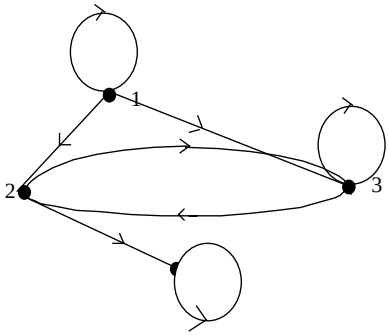


22. The expression tree below represents which expression?



A: ((4**x*) -6)+ (5/3)
B: ((*x*-4) *6) +(5/3)
C: (*x*-4) *6- (5/3)
D: (4**x**6-(5/3))

23. Which adjacency matrix corresponds to the directed graph below?



A: $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ **B:** $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ **C:** $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ **D:** $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

24.

If A = $\begin{bmatrix} 2 & 1 \\ -1 & 0 \\ 3 & 4 \end{bmatrix}$ and B = $\begin{bmatrix} 4 & 1 \\ 6 & -1 \\ 1 & 3 \end{bmatrix}$

then A - B is

A: $\begin{bmatrix} 6 & 2 \\ 5 & -1 \\ 4 & 7 \end{bmatrix}$ **B:** $\begin{bmatrix} -2 & 0 \\ -7 & 1 \\ 2 & 1 \end{bmatrix}$ **C:** $\begin{bmatrix} -2 & 0 \\ 7 & 1 \\ 2 & -1 \end{bmatrix}$

D: not possible to evaluate.

25. Which of these matrix expressions can **not** be evaluated:

A: $\begin{pmatrix} 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ **B:** $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

C: $\begin{pmatrix} 1 \\ 3 \end{pmatrix} \begin{pmatrix} 2 & 2 & 1 \end{pmatrix}$ **D:** $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$

CMSCD1006 Examination Semester 1 1999

Multiple Choice Answer Sheet (to be handed in)

Name:	
Registration No:	

Question	**Answer (enter either A or B or C or D only)**
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Laws of Logic

1. Commutative: $p \wedge q \Leftrightarrow q \wedge p$ and similarly for $\vee$ and $\Leftrightarrow$

2. Associative: $p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$ and similarly for $\vee$

3. Distributive: $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$ and similarly for $\vee$ and $\wedge$ interchanged

4. De Morgan's laws: $\neg(p \wedge q) \Leftrightarrow (\neg p \vee \neg q)$ and similarly for $\vee$ and $\wedge$ interchanged.

5. Negation: $\neg\neg p \Leftrightarrow p$

6. Excluded middle: $p \vee \neg p \Leftrightarrow T$

7. Contradiction : $p \wedge \neg p \Leftrightarrow F$

8. Implication: $p \Rightarrow q \Leftrightarrow \neg p \vee q$

9. Equality: $(p \Leftrightarrow q) \Leftrightarrow (p \Rightarrow q) \wedge (q \Rightarrow p)$

10 Simplification: $p \vee p \Leftrightarrow p$ $\quad p \vee T \Leftrightarrow T$
$p \vee F \Leftrightarrow p$ $\quad p \vee (p \wedge q) \Leftrightarrow p$

$p \wedge p \Leftrightarrow p$ $\quad p \wedge T \Leftrightarrow p$ $\quad p \wedge F \Leftrightarrow F$
$p \wedge (p \vee q) \Leftrightarrow p$

11 Exclusive Or : $p \vee_e q \Leftrightarrow (p \wedge \neg q) \vee (\neg p \wedge q)$