Liverpool John Moores University
SCHOOL OF COMPUTING AND MATHEMATICAL SCIENCES

Semester 2 Examinations 2000

CMSMA3016

Computational Mathematics

Duration 2 Hours

Instructions to candidates

Do not open this question paper until you have been told to do so by the invigilator.

The figure in [] denotes the number of marks available for that question or part of question.

There are 6 questions. **Answer 3 questions.**

Questions carry 25 marks each. The total number of marks available is 75.

You have the use of the Software DERIVE 5™. You may also use any Derive functions that you have developed throughout the course. You may not use a printer to print out any mathematical expressions or graphs.

Any results or graphs that DERIVE produces, on screen, that are relevant to your examination work should be written or sketched in your answer book.

You may also use **any** calculator and its memory need **not** be erased before the examination.

The sending or the reading of email and use of the internet/intranet during this examination is prohibited.

1.

(a) Suppose that your computer is storing a decimal number into a 32-bit word using the IEEE Single Precision Floating Point Standard. Describe how this is done. Be sure to identify all fields within the word and how the information is stored in each field.

[7]

(b) Write the decimal number 0.625 = $\frac{1}{2}+\frac{1}{8}$ in IEEE Single Precision Floating Point Format.

[3]

(c) Explain (you do not need to produce a result) how the computer would multiply .625 by 2.5.

[5]

(d) What is the advantage over a signed/magnitude system of storing negative integers in two's complement format?

[5]

(e) Why does two's complement form produce one more negative integer than positive integer?

[3]

(f) Is it possible to have a system of binary integers that has a unique representation for every integer in its range and also has as many positive integers as negative integers? Explain.

[2]

Total [25]

2.

(a) Fermat's Little theorem states that:

If p is a prime number then $a^{p-1} \bmod p = 1$*, where a is an integer that is* ***not*** *a multiple of p.*

Use Fermat's Little Theorem to prove that 4294967295 is **not** a prime number. [3]

(b) Explain why Fermat's Little Theorem cannot be used to prove or disprove that 4294967297 is prime using $a = 2$. Find a value for a that proves that 4294967297 is in fact not prime. [5]

(c)

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
⋮	⋮	⋮	⋮	⋮	⋮

Use the table above to demonstrate that the series

$$x_{n+1} = \begin{cases} x_n + 2 & \text{if } n \text{ is odd} \\ x_n + 4 & \text{if } n \text{ is even} \end{cases} \quad \text{where } x_1 = 5$$

will generate a list of numbers that *includes* all the prime numbers greater than 3. [5]

(d) Describe the method of trial division to determine whether n is a prime number, explaining why only numbers below $\sqrt{n}$ need be considered in the trial division. [6]

(e) Using trial division, prove that 1361 is a prime number. Similarly, use trial division to prove that 1357 is not a prime number. [6]

Total [25]

3.

(a) Explain why it is not practical to use the Taylor series for $\ln(1+x)$, about the point $x=0$, to evaluate $\ln z$ for all $z>0$. [2]

(b) Suggest an alternative function whose Taylor series can be used to evaluate $\ln z$ for $0.5<z\leq 1$. Stating why this function is better than $\ln(1+x)$. [3]

(c) Use your *alternative* Taylor series to evaluate $\ln 2$ to correct 6 significant figures. [4]

(d) Describe how you would efficiently find the value of $\ln 1000$ using Taylor series. Hence, evaluate $\ln 1000$ correct to 6 significant figures. [5]

(e) Using the definition $\sinh x = \dfrac{e^x - e^{-x}}{2}$, describe how $\sinh^{-1}(x)$ can be evaluated using the functions $\ln(x)$ and $\sqrt{x}$. [4]

(f) Hence, evaluate $\sinh^{-1}(2)$ using only the functions $\ln(x)$ and $\sqrt{x}$. [2]

(g) Given that you have the capability to evaluate square roots, evaluate $\sinh^{-1}(1000)$ to 6 decimal places using the Taylor series of $\ln$ discussed in part (d). [5]

Total [25]

4.

(a) Given that Chebyshev polynomials are generated from the recurrence equation

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x), \text{ where } T_0(x) = 1, \quad T_1(x) = x$$

evaluate $T_5(x)$.

[4]

(b) Find, or write down, the 5th order Taylor series of $\sin(x)$ about $x = 0$. Using $T_5(x)$, find the *economised* third order polynomial that approximates $\sin x$. [5]

(c) Compute the error in approximating $\sin(x)$ with the 5th degree Taylor series and the 3rd order Chebyshev economised polynomial, when *x* takes the values 0.2, 0.4, 0.6 & 0.8. [4]

(d) What are the two advantages to using a Chebyshev economised polynomial, as opposed to the Taylor series? [3]

(e) Use the Padé method to approximate $\sin x$ as a rational function

$$\frac{a + bx + cx^2 + dx^3}{1 + ex + fx^2}$$

where a, b, c, d, e & f are to be determined. Use this rational function to approximate $\sin(0.4)$. [9]

Total [25]

5.

Define

$$A = \begin{pmatrix} -1 & 6 & -4 \\ -5 & 0 & 2 \\ 1 & 1 & 0 \\ 0 & -1 & 8 \\ 2 & 1 & 2 \end{pmatrix}$$

(a) Write out this sentence and fill in the blanks

The range of A is a space of at most dimension ____ located within a space of dimension ______ .

[2]

(b) Find an ortho-normal basis for the column space of A using the Gram-Schmidt process. You must show all the numerical steps.

[8]

(c) Find an upper triangular matrix, R, such that A = QR.

[5]

(d) Why does the procedure you used in (c) *always* produce an upper triangular matrix.

[10]

Total [25]

6.

(a) Describe in detail how you would perform the QR method to find eigenvalues of any *n* x *n* matrix. [12]

You will need to use either Derive 5 or MATLAB to attempt the remainder of this question. If you use Derive you will need to load the Derive file QR.DFW that is held in the directory L:\MA3016. Write all numbers to 6 significant figures.

(b) Given the Matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 0 & 9 & 8 & 7 \\ 0 & 0 & 6 & 5 \end{bmatrix}$

Find the orthogonal matrix Q and the upper triangular matrix R where $A = QR$. [2]

(c) Apply the QR method once (no shift) and write down the ensuing matrix. [2]

(d) Apply the QR method iteratively 20 times and write down one of the estimates of the eigenvalues of A. [4]

(e) Now find the other three eigenvalues of A. [5]

Total[25]